A Note on the Design of the Gap Between Helices on a Double-Helical Gear

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Abstract

It is readily observed that the design of the gap between helices on a double-helical gear and the design of the manufacturing process(es) employed to manufacture the gear are inseparable. In particular, the outside diameter and swivel angle of a hob impose a necessary lower bound on the gap between the helices it is meant to produce. There are a number of different simple formulas meant to give approximations to this lower bound. Most are claimed to be accurate within a certain degree of precision. The purpose of this article is to illustrate an analytical solution to the problem that achieves arbitrarily small error.

1 Introduction

It is readily observed that the design of the gap between helices on a double-helical gear and the design of the manufacturing process(es) employed to manufacture the gear are inseparable. In particular, the outside diameter and swivel angle of a hob impose a necessary lower bound on the gap between the helices it is meant to produce. There are a number of different simple formulas meant to give approximations to this lower bound. For instance, Dudley's famous Handbook of Practical Gear Design and Manufacture gives the formula

$$\sqrt{h(D_H - h)}\cos\Psi_1 + \frac{n_1 p_n \sin\Psi_1}{\cos\lambda} + \frac{x' \sin\Psi_1}{\tan\phi_n},$$

where h is the depth of cut, D_H is the hob outside diameter, Ψ_1 is the swivel angle of the hob, λ is the lead angle of the hob, p_n is normal circular pitch,

n_1 is the number of pitches from hobbing center, x' is the maximum of the gear addendum and the gear dedendum minus clearance, and ϕ_n is the normal profile angle. The claim is made that the formula is usually accurate within plus or minus 5%. It is stated that an exact calculation of gap width is quite difficult.

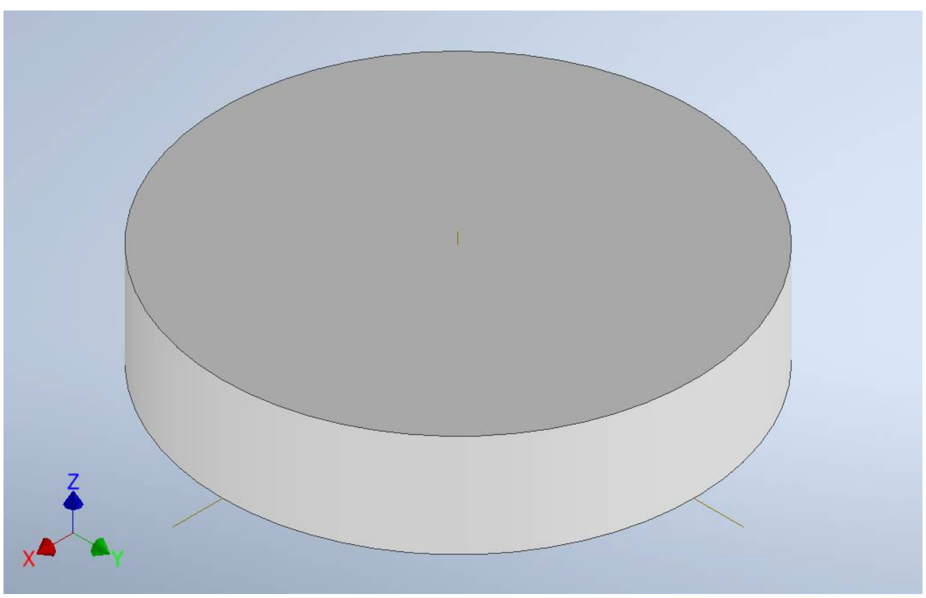
The purpose of this article is to illustrate that the calculation is not so bad. In fact, it is a cute exercise in numerical vector analysis. Note that there are a number of software packages available that perform this computation. The underlying geometric complexity is what we wish to reveal.

2 The Solution

We may suppose that the axis of a gear in the traditional xyz-coordinate system coincides with the z-axis, and that the outside diameter cylinder is the set of solutions to the equation

$$x^2 + y^2 = \left(\frac{D_o}{2}\right)^2,$$

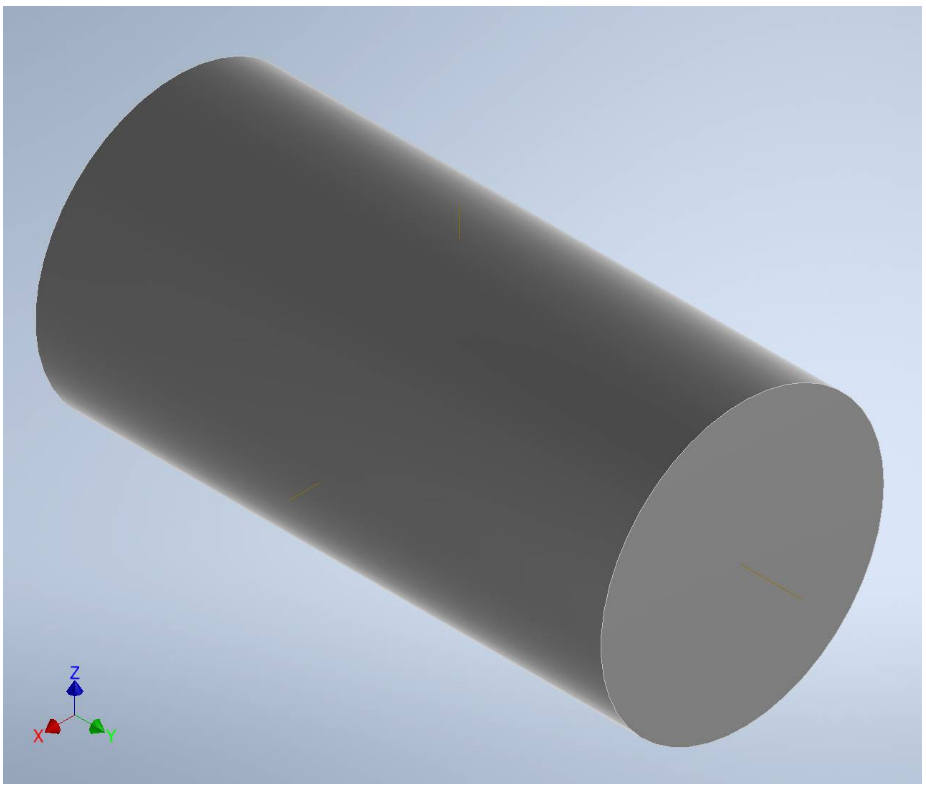
where D_o is the outside diameter of the gear.



We may further suppose that the axis of a hob coincides with the y-axis, and that the outside diameter cylinder is the range of the parametrization

$\mathbf{r}_1 : \mathbb{R} \times [0, 2\pi) \to \mathbb{R}^3$ given by

$$\mathbf{r}_1(u,\theta) = \begin{bmatrix} \dfrac{D_H}{2}\cos\theta \\ u \\ \dfrac{D_H}{2}\sin\theta \end{bmatrix}.$$



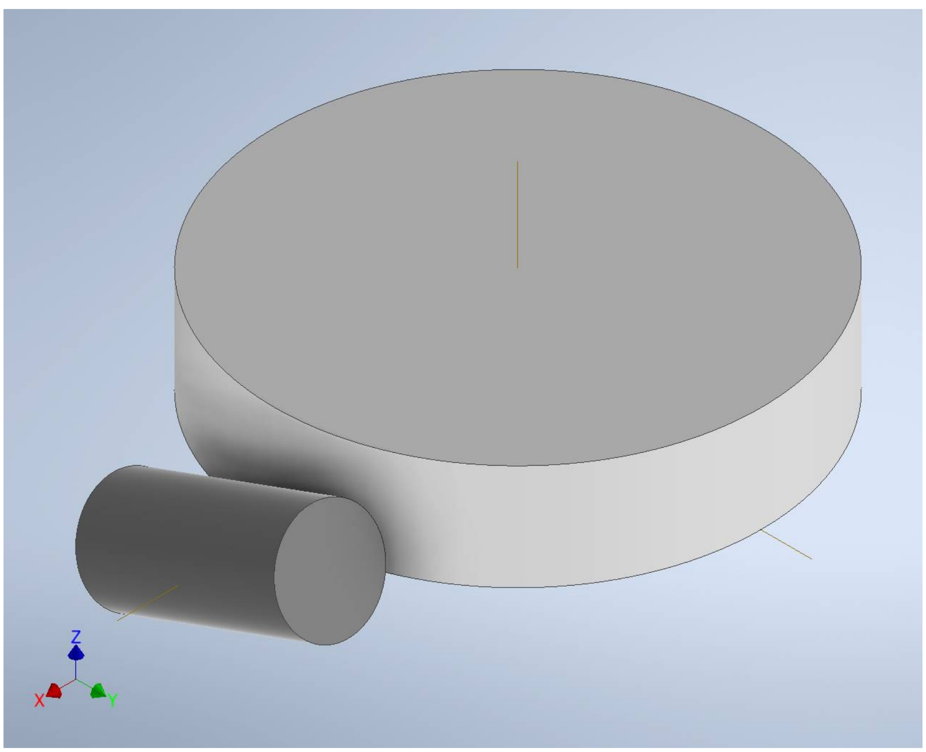
Now, the matrix representation, under the standard basis, of the linear transformation that rotates $\mathbb{R}^3$ around the x-axis by the angle Ψ_1 is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Psi_1 & -\sin\Psi_1 \\ 0 & \sin\Psi_1 & \cos\Psi_1 \end{bmatrix}.$$

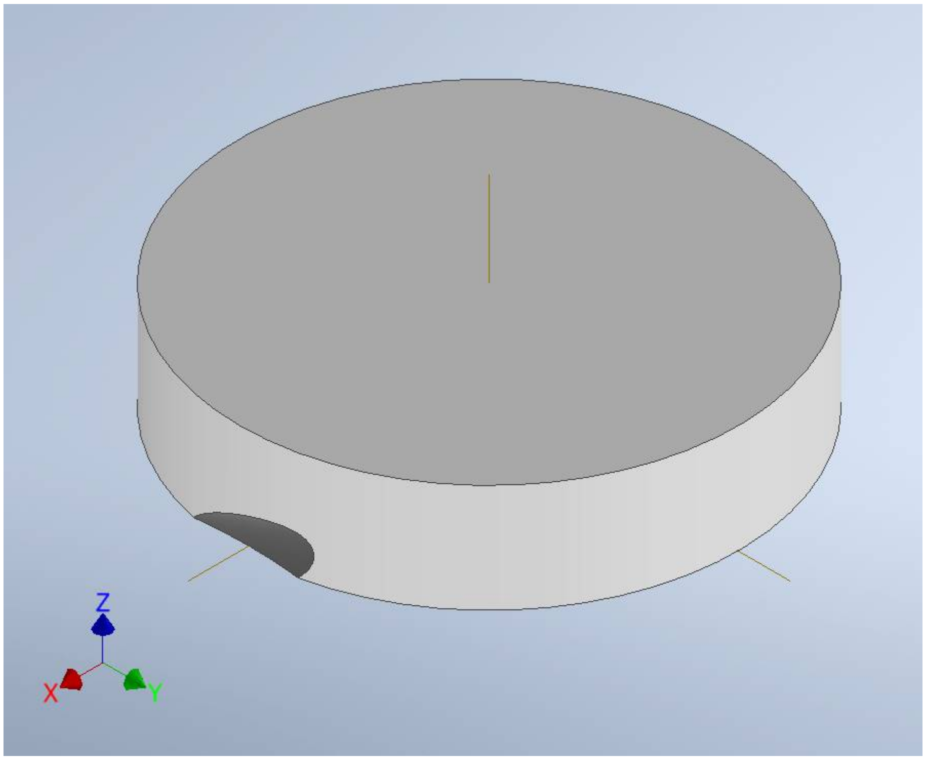
Letting $E = \dfrac{D_o}{2} + \dfrac{D_H}{2} - h$ be the center distance of the gear and the hob, we may compose the rotation and the translation to construct the parametrization $\mathbf{r} : \mathbb{R} \times [0, 2\pi) \to \mathbb{R}^3$, given by

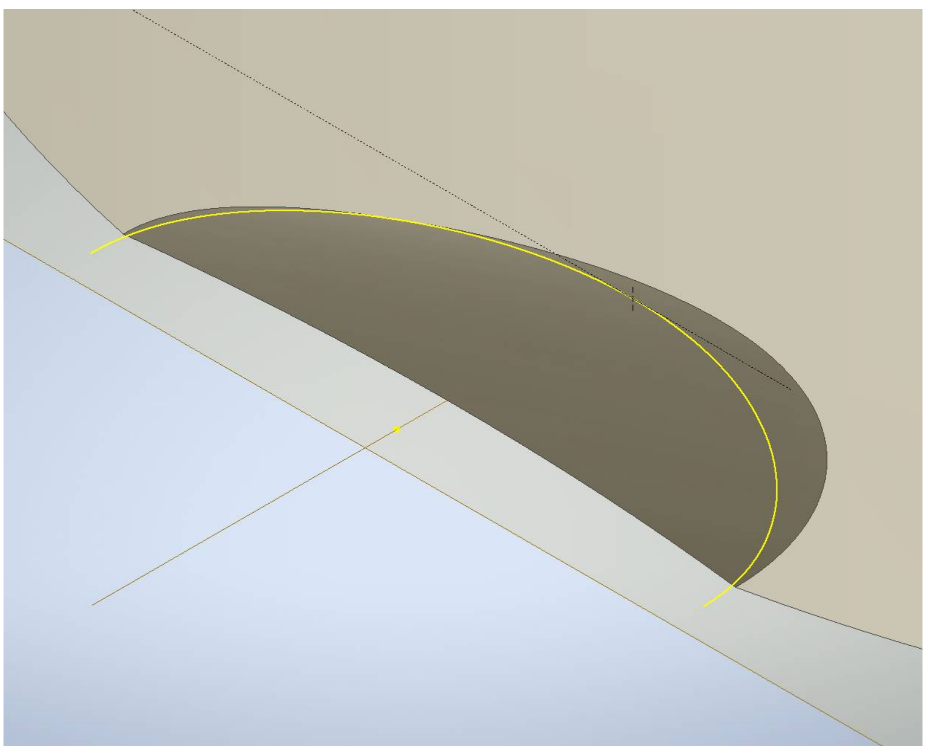
$$\mathbf{r}(u,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Psi_1 & -\sin\Psi_1 \\ 0 & \sin\Psi_1 & \cos\Psi_1 \end{bmatrix} \begin{bmatrix} \frac{D_H}{2}\cos\theta \\ u \\ \frac{D_H}{2}\sin\theta \end{bmatrix} + \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{D_H}{2}\cos\theta + E \\ u\cos\Psi_1 - \frac{D_H}{2}\sin\Psi_1\sin\theta \\ u\sin\Psi_1 + \frac{D_H}{2}\cos\Psi_1\sin\theta \end{bmatrix},$$

of the outside diameter cylinder of the hob in the fixed coordinate system of the gear cutting machine with the full engagement of the hob running along the x-axis.



With the full engagement of the hob occurring where $z = 0$, we may find the required gap by computing the largest value of z appearing in the intersection of the two surfaces.





In other words, we have reduced the problem to finding the point on the cylinder $x^2 + y^2 = \left(\frac{D_o}{2}\right)^2$ that maximizes $z = u \sin \Psi_1 + \frac{D_H}{2} \cos \Psi_1 \sin \theta$.

Equivalently, we must optimize the function

$$f(u,\theta) = u\sin\Psi_1 + \frac{D_H}{2}\cos\Psi_1\sin\theta,$$

subject to the constraint

$$g(u,\theta) = \left(\frac{D_H}{2}\cos\theta + E\right)^2 + \left(u\cos\Psi_1 - \frac{D_H}{2}\sin\Psi_1\sin\theta\right)^2 - \left(\frac{D_o}{2}\right)^2 = 0.$$

Using g to eliminate the variable u in f, we see that f takes on the form

$$\Phi(\theta) = \tan\Psi_1\sqrt{\left(\frac{D_o}{2}\right)^2 - \left(\frac{D_H}{2}\cos\theta + E\right)^2} + \frac{D_H}{2}\sec\Psi_1\sin\theta.$$

Here we are making the restriction

$$\theta \in \left[\arccos\left(\frac{2h}{D_H} - 1\right), 2\pi - \arccos\left(\frac{2h}{D_H} - 1\right)\right]$$

so that the square root is well-defined.

We may then compute

$$\Phi'(\theta) = \frac{D_H\tan\Psi_1}{2}\sin\theta\frac{\frac{D_H}{2}\cos\theta + E}{\sqrt{\left(\frac{D_o}{2}\right)^2 - \left(\frac{D_H}{2}\cos\theta + E\right)^2}} + \frac{D_H}{2}\sec\Psi_1\cos\theta.$$

In all practical circumstances, we may observe that

$$\lim_{\theta\to\arccos\left(\frac{2h}{D_H}-1\right)^+}\Phi'(\theta) = +\infty,$$

$$\Phi'(\pi) = -\frac{D_H}{2}\sec\Psi_1 < 0,$$

and that $\Phi'(\theta)$ is strictly decreasing on the interval $\left(\arccos\left(\frac{2h}{D_H} - 1\right), \pi\right]$. We may then deduce that there exists a unique root, say α, in said interval and employ our favorite numerical root-finding algorithm to find it, concluding that $\Phi(\alpha)$ is the prescribed minimum gap.

3 Remarks

- Forest City Gear has not found it necessary to use anything more sophisticated than the famously robust bisection method for numerically finding the root of this function on this interval.

- Keep in mind that this is a theoretical solution of a purely geometric problem. One must also take into account the overtravel of the hob and the clearance between the hob and the opposing helical gear the manufacturer requires during machining. This is dependent on a number of factors, including the manufacturer, the precision of the blanks, the precision of the workholding, and the volume of parts being produced. Feel free to contact the author for consultation on your next gear manufacturing project.